# Introduction to NMR Algorithm

## Metaheuristic optimization algorithm

Nature has always served as a problem solver and it has been solving real complex problems for millions of years. A large number of natural systems are working in collaboration to effectively achieve numerous tasks at hand. The mammalian body itself consists of a large number of systems working in a systematic way for performing various body functions, eyes as a perfect visual system, ears as a sound `receptor, brain as the mastermind of all systems and others. Animal species such as wolves show predator-prey system, honey bees showing cooperative behavior, fireflies attracting other fireflies for mating, moths moving toward the source of light, whales searching for preys in water. In short, all natural species show some sort of intelligence and artificial systems have been derived using their knowledge from the past two decades. The knowledge base has itself been developed into a separate field of study and a large number of algorithms known as nature-inspired algorithms have been derived. Nature-inspired algorithms are becoming extremely popular for solving various optimization problems. This is due to the fact that these algorithms search for the fittest solution based upon ‘trial and error’ criterion. Also, they are easy to implement due to their simple conceptual model and the east requirement of gradient information. Broadly, these algorithms have been inspired by theories like Darwin’s theory of natural selection and social behavior of living organisms. Darwin’s theory led to the development of evolutionary algorithms (EAs), while social behavior paved the way for swarm intelligent (SI) algorithms. All EAs are stochastic and hence require a minimum pool of population to improve their performance while for SIs new solutions are generated by incorporating appropriate operators, like the crossover, mutation, probability, and others. Generally, EAs randomly select a population and evolve it over subsequent iterations. The best individuals of each iteration are combined together to form a new solution and hence optimizing all the solutions over subsequent iterations. These algorithms are best fit for solving problems where no initial experience is required and even without any specific knowledge about the problem under test. This is the reason that these algorithms have been applied to almost every domain of optimization. The examples of these algorithms are genetic algorithm (GA) [1], differential evolution (DE) [2], evolutionary strategy (ES) [3], biogeography-based optimization (BBO) [4], genetic programming (GP) [5], ant lion algorithm (ALO) [6] and probability based incremental learning (PBIL) [7]. The second group of nature-inspired algorithms is SI. Swarm in itself means the group of different organisms, such as ants, birds, bees, monkeys, and wolves, which work collectively towards a common goal. SIs use the learning ability and adaptive nature of these organisms to solve really complex problems at hand. The first work on SI was reported by Beni and Wang [8] in cellular robotics. The field has flourished from that day and today, these algorithms have been applied to in almost every field whether it is electrical, management, electromagnetics, structural engineering design, and others. The most recent algorithms which have been applied to a lot of problems include, artificial bee colony (ABC) [9], particle swarm optimization (PSO) [10], firefly algorithm (FA) [11], bacteria foraging optimization (BFO) [12], flower pollination algorithm (FPA) [13, 14], bat algorithm (BA) [15] and so on. Other recently introduced algorithms include salp swarm algorithm [16], dragonfly lgorithm [17], cuckoo search [18, 19], hybrid GA [20], water wave optimization [21], whale optimization algorithm [22], moth flame optimization [23], and others.

## The DE Algorithm

For the first iteration basic steps are imlpementment in the classic DE including (i), mutation, (iii) crossover, and (iv) selection. While only the last three steps are repeateddly done in subsequent iteration until satisfying stopping criteria which depend on each individual optimization problem are defined by users. A brief introduction to the above steps is provided in detail below.

### Initialization

An optimization problem including np individual is randomly initialized in a given continuous search space. In which, the ith individual ( i= 1,2,…,np) is a vector consiting of d design variables and is given by the following from

, j = 1,2,…,d

Where is a uniformly distributed random number within and is independently created for each term in the ith vector; and  are the predefined minimum and maximum bounds of , respectively. Note that the supersript (t=0) stands for  at the frist iteration. Generally, let us adopt the following notation for defining the ith individual vector at the t current iteration (t = 0,1,…,tmax ),



### Mutation

After the frist step, a mutant vector  is generated from the target vector at the current iteration via mutantion. Five most commonly utilized mutantion scheme are given aas follws

rand/1: 

best/1: 

current – to – best/1: 

rand/2: 

best/2: 

where R1,R2,R3,R4 and R5 are the randomly chose integer numbeer within in such a way that all of those are totally diffetent from the index I; the scale factor F is randomly selected in to control the deviation from tow tager vectors, and  is yeh best individual vector corresponding to the best objective function at the current iteration.

It ca be seen the obtained by one of the above mutantion opertors may be violated its lower and upper bounds. Therefore, in order to strictly satisfy the boundary constraints, it is returned to the rearch space by the following formula



### Crossover

Next, crossover scheme in used to enchance the diversity of individual vector in the current population. According to this scheme, the ith trial vectoris produced by mixing the tager vector and the mutant vector as



Where K is an integer number randomly chosen in , and Cr is the crossover control parameter in .

### Selection

This step aims to select better indivdual in the current population for the performace of the next iteration be comparing each pair pf the objection function values f(.) for the individual vectors of and . This strategy can be expressed as



## The NMR Algorithm

### Naked Mole Rat

Sexually reproducing animals are evolving mainly due to inbreeding, this is due to the fact thatbsexual recombination helps to increase the variability of offspring’s. This helps to increase efficiency, remove deleterious alleles from different genes and provide resistance from parasites .This also helps species in adapting to local conditions. There is a type of inbreeding called close inbreeding which occurs when alternative males are not available. This type of phenomenon is very rare in natural species. Fallow deer is one such animal showing a father-daughter mating and has been confirmed by the genetic model .But from the behavioral and paternity analysis, there is no conformity of such behavior . The only other species showing inbreeding patterns is the NMR.

NMRs occur in semi-arid eastern Africa and are fossorial, showing large subterranean tunnel systems while foraging for tubers as shown in Fig. 2 b. They have evolved and adapted to this hostile habitat and are found to be present on this earth from the Miocene era (24 million years ago) . NMRs were firstly described in African mammals by Eduard Ruppell in 1842. He named them ‘different headed’ (Heterocephalus) and smooth-skinned/hairless (grabber) because of their odd shaped skull and absence of flurry pelage as shown in Fig. 2 a. They belong to the kingdom: Animalia, phylum: chordate, order: Rodentia, family: Bathergidae and was the only species classified in genus Heterocephalus until the 1980s. But till date, 15 species have been included in this genus . NMRs as discussed, are close inbreeding animals and they do so in a belief that new colonies can be formed through fission, helping in sealing the tunnels .By sealing the tunnels, the danger due to predation, extreme temperatures, and starvation, are minimized. The subsequent subsections provide a brief overview of NMR behavior.

A picture containing rodent, mammal

Description automatically generated

### Mating patterns in NMR

They are eusocial animals living in a group of up to 295 individuals. A detailed discussion about eusociality in NMR is given in the next subsection. The average population of a single colony is through close to 75. The colony is headed by a single female, that is, there is only a single breeding female called queen. This queen breeds with a limited number of males, while the remaining members of the colony perform necessary tasks like construction, provisioning, maintenance, and defense. Workers are sterile and become sexually active only when separated from the colony. They separate from the colony only to act in the assistance of offspring produced by the queen . So, based upon the mating patterns, we can define two types of NMRs that are breeders (assist in mating) and workers (helping to perform other tasks). Breeders, live close to the queen and hence are at no risk or at lower risks of predation while workers are not. This is the reason why breeders live up to 17 years, 4 times longer than their nonbreeding counterparts or workers . So, is there only a single female in the colony that grows to a queen, if so, how will the new queen be selected if present one die? The answer is that there are a large number of females in the colony and they form the worker class. Any female who is older than 6 months is capable of becoming a queen. This further adds a question mark. Where will the present queen go, if any female can be queen? All the females in the group may fight until death to become the queen and establish their dominance . When a female becomes the queen, it becomes reproductively active and starts producing estrogen-dependent ‘pubescent’ growth surge resulting in an increase in their body lengths . The other females in the colony remain anovulatory with a lower level of sex steroids . Males in the worker groups exhibit the same patterns and have low testosterone, low luteinizing hormone, abnormal sperms and infertile . Hence, it can be said that only the best males among the worker class form the breeder class and one among the best females get the status of a queen. The breeding mates, breeders, and workers are shown in Fig. 3.

A picture containing graphical user interface

Description automatically generated

#### Eusociality in NMR

Eusociality is defined as the colonial lifestyle of a species, following a strict division of labor and having a single female for breeding. This concept was laid down by Richard Alexander in the mid 1970s. He identified that such systems are found in the subterranean population living in soil and relying on others to dig burrows and forage food for underground tubers [51]. This theory was explored by Jarvis, who found that like termites, ants and wasps carry out tasks in a single community of about 295 individuals, with 1 to 3 breeding males and only a single breeding female [52]. So, it is evident from the study that NMRs are eusocial animals following the division of labor to perform various tasks and having only a single female called queen for mating. Division of labor is followed only by the workers whereas breeders and queen are only meant for reproduction. This eusocial lifestyle of NMRs improve their inclusive fitness and ultimately contribute to the prolonged lifespan.

#### NMR and SI

Since the main focus of the paper is to design a new SI algorithm, so it is necessary to find out if that particular species is SI or not. In this subsection, we will try to find out if there is any close association of SI with NMR. So, if NMR will follow the necessary conditions of self-organization and division of labor, we can count it as SI animal. As explained:

A. Self-organization: It means a species should produce a global level response from low-level interactions without any central authority. In NMR, the workers act as a low-level component and they produce high-level breeder. These high-level breeders further choose among themselves the best breeder, who mates with the queen. So, a hierarchical pattern is followed without any central authority. Hence, we can say that NMRs follow self-organization. This can be further validated by proving the four steps of self-organization as:

1. Positive feedback: In NMR, the queen is assisted by a few sets of breeding males and only they contribute to the reproduction process. These breeding males are selected from the pool of worker population. This process from workers to breeders and ultimately to queen follows positive feedback. This helps in providing diversity and accelerating the system to move to a new stable state.
2. Negative feedback: This part focuses on compensation through positive feedback. The breeder who is not able to cooperate or whose fitness is not up to the mark for mating,either dies or is sent back to the worker’s pool, hence forming a negative feedback path from breeders to workers. This process stabilizes the system and hence maintain a proper level of breeder population.
3. Fluctuations: It means randomness in swarms. In NMR, randomness is in finding the best possible breeders and queen among the workers. This process helps in getting rid of the stagnation problem.
4. Multiple Interactions: It is a learning process either by interactions or experience. Here, the worker pool of NMRs interact with each other and cooperate or assist each other in completing various tasks like maintenance, construction, defense and provisioning. The breeders also follow this pattern by cooperating with each other during mating, as only a single male with a mate with the queen at a particular time and hence enhance the combined intelligence of all NMRs.

B. Division of labor: It has been already discussed that NMRs are eusocial animals and every eusocial animal follows the strict division of labor. Here the workers basically do all sorts of work and breeders do the mating.

From the above discussion, it is evident that NMRs are SI animals. In the next section, a new SI based algorithm called NMRA has been proposed.

### Naked Mole Rat Algorithm

The social behavior of NMR has inspired authors to propose a stochastic optimization algorithm. The algorithm mimics the mating patterns in NMR and has the following key features:

1. NMRs are eusocial animals living in a group of 295 members with an average number of members to be 70-80. For experimental analysis, the present work uses a group of 50 NMRs.
2. A female queen leads the group and divides the population among breeders and workers. The breeders are the best performing NMR among the working group and are meant for mating only whereas workers perform other tasks.
3. The workers perform necessary tasks and best among them are replaced by breeders. In simple words, high performing workers, become breeders and low performing breeders are again sent to the worker’s pool.
4. The best breeder among the breeder pool mates with the queen.  
   The above four rules have been idealized to propose a naked mole rat algorithm (NMR). The algorithm is divided into three phases. In the first phase, the population of NMRs is initialized, second is the worker phase and third is the breeder phase. The breeder phase is selected based upon the breeding probability. The details of the algorithm are explained as:

#### Initialization:

Initially, it generates a uniformly distributed random population of n NMR where each NMR in the range [1, 2 … n] is a D-dimensional vector. Here D epresents the number of variables or parameters to be tested in the problem. Each NMR is initialized as

𝑁𝑀𝑅𝑖,𝑗 = 𝑁𝑀𝑅𝑚𝑖𝑛,𝑗 + 𝑈(0,1) × (𝑁𝑀𝑅𝑚𝑖𝑛,𝑗 - 𝑁𝑀𝑅𝑚𝑎𝑥,𝑗) (1)

where 𝑖 ∈ [1, 2, … 𝑛], 𝑗 ∈ [1, 2, … 𝐷], 𝑁𝑀𝑅𝑖,𝑗 is the 𝑖𝑡ℎ solution in the 𝑗𝑡ℎ dimension,  
𝑁𝑀𝑅𝑚𝑖𝑛,𝑗, 𝑁𝑀𝑅𝑚𝑎𝑥,𝑗 are the lower and upper bounds of the problem function respectively and 𝑈(0,1) is uniformly distributed random number. After initialization, the objective function is evaluated and its fitness is calculated. Based upon the fitness, B breeders and W workers are identified and overall initial best solution d is calculated. After initialization, the population of NMR is subjected to epeated cycles or iterations of the search process of worker and breeder phase.

#### Worker phase:

In this phase, the workers tend to improve their fitness so that they get a chance to become a breeder and eventually mate with the queen. So here, the new solution of worker NMR is generated based upon its own experience and local nformation. Here the fitness of new NMR is evaluated and if the new mating fitness is better, the old solution is discarded and the new solution is memorized. Otherwise, the older solution is retained. After all the worker rats complete the search process, the final fitness of all of them is remembered. In order to produce a new solution from the old one, the NMR uses the following equation:

𝑤𝑖𝑡+1 = 𝑤𝑖𝑡 + 𝜆(𝑤𝑗𝑡 - 𝑤𝑘𝑡 ) (2)

where 𝑤𝑖𝑡 corresponds to the ith worker in the tth iteration, 𝑤𝑖𝑡+1 is the new solution or worker, 𝜆 is the mating factor and 𝑤𝑗𝑡 and 𝑤𝑘𝑡are two random solutions chosen from the worker's pool. The value of 𝜆 is obtained from a uniform distribution in the range of [0, 1].

#### Breeder phase:

The breeder NMR also update themselves in order to be selected for mating and also to stay as a breeder. The breeder NMRs are updated based upon a breeding probability(𝑏𝑝) with respect to the overall best d. This 𝑏𝑝 is a random number in the range of [0, 1]. Some of the breeders may not be able to update their fitness and hence may be pushed back to the workers’ category. The breeders modify their positions according to the equation given below:

𝑏𝑖𝑡+1 = (1 - 𝜆)𝑏𝑖𝑡 + 𝜆(𝑑 - 𝑏𝑖𝑡) (3)

Here 𝑏𝑖𝑡 corresponds to the breeder i in the iteration t, 𝜆 factor controls the mating frequency of breeders and helps in identifying a new breeder 𝑏𝑖𝑡+1 in the next iteration. To start with, the value of 𝑏𝑝 has been set to 0.5 as the initial value.

For simplicity, we have assumed that there is only a single queen and best among the breeder mates with the queen. So here we find only the best breeding male who will breed with the female. The algorithm works by differentiating or identifying the breeders and workers among the pool of NMRs. After an initial evaluation, the best breeder, and the best worker is selected. The fitness of workers is updated so that their fitness improves and they may get a chance to become breeders. On the other hand, breeders also update their fitness based upon breeding probability so that they remain breeders. The breeder which becomes sterile will be pushed into the workers’ category. The best breeder among the population serves as the potential solution to the problem under test. The above-mentioned workerbreeder relationship and their mating with the queen are best elaborated in Fig. 4. The pseudo-code for NMRA is given in Algorithm 1

***Begin:  
Inputs:*** *Initialize naked mole rats: n  
breeders* 𝑩: 𝒏/𝟓  
*workers* 𝑾: 𝑩 - 𝒏  
*define breeding probability:* 𝒃𝒑  
*define* ***D****-dimensional objective function,* ***f(x)  
Output:*** *find the overall best* 𝒅  
***do Until*** *iteration < maximum number of iterations****for i=1: W****perform worker phase:* 𝒘𝒊 𝒕+𝟏 = 𝒘𝒊 𝒕 + 𝝀 (𝒘𝒌 𝒕 - 𝒘𝒋 𝒕 )  
*evaluate* 𝒘𝒊 𝒕+𝟏  
***end for  
for i=1:B  
if*** 𝑼(𝟎, 𝟏) > 𝒃𝒑  
*perform breeder phase:* 𝒃𝒊 𝒕+𝟏 = (𝟏 - 𝝀)𝒃𝒊 𝒕 + 𝝀(𝒅 - 𝒃𝒊 𝒕)  
*evaluate* 𝒃𝒊 𝒕+𝟏  
***end for****combine the new worker and breeder population  
evaluate the population  
update the overall best* ***d****update iteration count****end until****save the final best (****d****)****End***

# Benchmark function

## Ackley’s Function



Subject .The gobal minimum is located at origin  with .

## Alpine Function



Subject .The gobal minimum is located at origin  with .

## Bartels Conn Function



Subject .The gobal minimum is located at origin  with .

## Brid Function



Subject to.The gobal minimum is located at origin with.

## Bohachesky Function



Subject to.The gobal minimum is located at origin  with .

## Booth Function



Subject to.The gobal minimum is located at origin  with .

## Brent Function



Subject to .The gobal minimum is located at origin  with 

## Brown Function



Subject to.The gobal minimum is located at origin  with 

## Bukin Function



Subject toand .The gobal minimum is located at origin  with 

## Cosine Mixture Function



subject to . The global minimum is located at  with   
for *D* = 2

## Csendes Function



subject to. The global minimum is located at with  
.

## Deb Function



Subject to . The number of global minima is that are evenly spaced in the function landscape.

## Exponential Function



Subject to . The global minimum is located at with  
.

## Griewank Function



Subject to . The global minimum is located at with  
.

## Hosaki Function



Subject to and .The gobal minimum is located at origin  with 

## Mishra Zero-Sum Function



Subject to . The global minimum is 

## Powell Sum Function



Subject to . The global minimum is 

## Quartic Function



Subject to . The global minimum is located at with  
.

## Quintic Function



Subject to . The global minimum is located at with  
.

## Salomon Function



Subject to . The global minimum is located at with.

## Schumer Steiglitz Function



The global minimum is located at with .

## Sphere Function



Subject to . The global minimum is located at with.

## Step Function



Subject to . The global minimum is located at with.

## Stepint Function



Subject to . The global minimum is located at with

## Sum Squares Function



Subject to . The global minimum is located at with.

## Styblinski-Tang Function



Subject to . The global minimum is located at with.

## Xin-She-Yang Second Function



Subject to.The gobal minimum is located at with

## Xin-She-Yang Fourth Function



Subject to . The global minimum is located at with

## Zakharov Function



Subject to . The global minimum is located at with

# Results and observations.

## Calculation results and comparison between DE and NMR Algorithm:

Table 1: Compairison between DE and NMR Algorithhm.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Function | Algorithm | Best | Worst | Mean | Standard Deviation |
| f1 | DE | 4.44E-16 | 0.010563348 | 0.000151 | 0.001098777 |
|  | NMR | 4.44E-16 | 4.44E-16 | 0.106703 | 0.83505378 |
| f2 | DE | 6.36E-15 | 0.08633093 | 0.001467 | 0.010222301 |
|  | NMR | 1.08E-16 | 8.99E-07 | 0.001467 | 0.010222301 |
| f3 | DE | 1 | 1.795857593 | 1.013681 | 0.085576635 |
|  | NMR | 1 | 1.795857593 | 1.835329 | 5.659354864 |
| f4 | DE | -106.7645367 | -105.9505756 | -106.749 | 0.094083299 |
|  | NMR | -106.7645367 | -106.6487378 | -106.276 | 2.868934935 |
| f5 | DE | 0 | 0.469882415 | 0.013391 | 0.074917041 |
|  | NMR | 0 | 0.469882415 | 0.161477 | 1.313547979 |
| f6 | DE | 0 | 0.114738313 | 0.001665 | 0.011799041 |
|  | NMR | 4.98E-11 | 0.357020128 | 0.618566 | 6.107017072 |
| f7 | DE | 1.38E-87 | 0.781176715 | 0.01072 | 0.081147342 |
|  | NMR | 6.80E-18 | 8.51E-08 | 0.391527 | 2.124197528 |
| f8 | DE | 4.40E-95 | 0.017307788 | 0.000175 | 0.001730679 |
|  | NMR | 1.52E-229 | 3.60E-08 | 0.000175 | 0.001730679 |
| f9 | DE | 4.65E-07 | 0.249933546 | 0.037635 | 0.051571633 |
|  | NMR | 1.32E-05 | 0.021122269 | 0.037635 | 0.051571633 |
| f10 | DE | -0.052215749 | -0.2 | -0.19839 | 0.014807628 |
|  | NMR | -0.2 | -0.2 | -0.19839 | 0.014807628 |
| f11 | DE | 2.44E-119 | 2.44E-119 | 3.63E-91 | 2.36E-90 |
|  | NMR | 0 | 0 | 3.63E-91 | 2.36E-90 |
| f12 | DE | -0.099998415 | -0.1 | -0.1 | 1.59E-07 |
|  | NMR | -0.1 | -0.1 | -0.1 | 1.59E-07 |
| f13 | DE | -1 | -1 | -1 | 0 |
|  | NMR | -1 | -1 | -1 | 0 |
| f14 | DE | 0.013161628 | 0.816441266 | 0.184846 | 0.16637577 |
|  | NMR | 0 | 0.816441266 | 0.184846 | 0.16637577 |
| f15 | DE | -2.34580502 | -2.260296709 | -2.34494 | 0.008550749 |
|  | NMR | -2.345811576 | -2.345811576 | -2.3435 | 0.013867276 |
| f16 | DE | 0 | 0.001210963 | 1.22E-05 | 0.00012109 |
|  | NMR | 0 | 0.001210963 | 1.22E-05 | 0.00012109 |
| f17 | DE | 0.001014801 | 0.052917181 | 0.003448 | 0.007385203 |
|  | NMR | 2.97E-106 | 2.97E-106 | 0.003448 | 0.007385203 |
| f18 | DE | 0.127341479 | 1.048814713 | 0.555405 | 0.232179663 |
|  | NMR | 6.37E-06 | 0.00303442 | 1.071997 | 1.133489347 |
| f19 | DE | 6.25E-06 | 66.44023536 | 7.584146 | 15.63311176 |
|  | NMR | 0.244731341 | 7.264277756 | 79.30734 | 184.5435152 |
| f20 | DE | 0.063649146 | 0.95491683 | 0.421637 | 0.275730924 |
|  | NMR | 0 | 0.95491683 | 0.775908 | 0.864755559 |
| f21 | DE | 2.380976648 | 253.3646613 | 100.6334 | 71.78262492 |
|  | NMR | 0 | 0 | 257.1624 | 316.5083289 |
| f22 | DE | 4.04455496 | 62.71692296 | 38.46405 | 15.93439002 |
|  | NMR | 5.49E-212 | 5.49E-212 | 63.68048 | 35.54080811 |
| f23 | DE | 33 | 124 | 89.25455 | 22.5245064 |
|  | NMR | 0 | 0 | 129.9798 | 55.16238284 |
| f24 | DE | -110 | -133 | -126.065 | 6.112717228 |
|  | NMR | 0 | 2 | -136.944 | 7.367135899 |
| f25 | DE | 98.85771262 | 456.167679 | 285.2613 | 89.45165395 |
|  | NMR | 4.26E-240 | 4.64E-175 | 463.2998 | 463.2997654 |
| f26 | DE | -64.19086206 | -75.81112538 | -66.8914 | 4.689029778 |
|  | NMR | -64.19561216 | -76.86206472 | -75.5047 | 5.655321082 |
| f27 | DE | 3.87E-119 | 0.32991479 | 0.059423 | 0.123766394 |
|  | NMR | 1.73E-118 | 2.77E-91 | 0.059423 | 0.123766394 |
| f28 | DE | 8.64E-12 | 1.75E-05 | 3.27E-06 | 4.39E-06 |
|  | NMR | 0.000123046 | -1 | 3.27E-06 | 4.39E-06 |
| f29 | DE | 6.62E-236 | 0.001023328 | 1.35E-05 | 0.00010547 |
|  | NMR | 9.25E-239 | 4.77E-191 | 1.35E-05 | 0.00010547 |

|  |  |
| --- | --- |
| F1: Ackley function | F2: Alpine function |
| F3: Bartels Conn Function | F4: Brid Function |
| F5: Bohachesky Function | F6: Booth Function |
| F7: Brent Function | F8: Brown Function |
| F9: Bukin Function | F10: Cosine Mixture Function |
| F11: Csendes Function | F12: Deb Function |
| F13: Exponential Function | F14: Griewank Function |
| F15: Hosaki Function | F16: Mishra Zero-Sum Function |
| F17: Powell Sum Function | F18: Quartic Function |
| F19: Quintic Function | F20: Salomon Function |
| F21: Schumer Steiglitz Function | F22: Sphere Function |
| F23: Step Function | F24: Stepint Function |
| F25: Sum Squares Function | F27: Xin-She-Yang Second Function |
| F28: Xin-She-Yang Fourth Function | F29: Zakharov Function |

## Observation:

After checking 29 Benchmark function for both DE and NMR Algorithm there are some observation in this report:

DE Algorithm has longer convergence time than NMR Algorithm.

Both DE and NMR Algorithm has some functions that have the different result such as: Sphere function, Hosaki function and stepint function.

Both Algorithm have some defect and need to be improved in the future.